Enhanced Knowledge in Sciences and Technology Vol. 2 No. 1 (2022) 221-230

EKST

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

Analysis On Hiemenz Flow Over a Shrinking Sheet in Hybrid Nanofluid

Muhammad Hafiezul Ridhwan Ibrahim[1], Fazlina Aman[1]*

[1]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,
Universiti Tun Hussein Onn Malaysia, 84600 Pagoh, Johor, MALAYSIA

*Corresponding Author Designation

DOI: https://doi.org/10.30880/ekst.2022.02.01.024
Received 02 January 2022; Accepted 02 March 2022; Available online 1 August 2022

Abstract: The Hiemenz flow of hybrid nanofluid across a shrinking sheet is investigated in this paper. The similarity equations are construct using similarity variables and then solved in Maple software using the shooting technique with RKF45 method. The effects of several governing parameters such as shrinking parameter and nanoparticles volume fraction for alumina oxide and copper on the flow behavior is examined and are depicted in table and graphs. It is found that both heat transfer rate and the skin friction on the surface escalated as cooper nanoparticles volume fraction increased. The opposite behavior is observed for alumina oxide nanoparticle volume fraction. Meanwhile, the velocity profiles decrease but the temperature profiles increase with shrinking parameter for fixed value of both copper and alumina nanoparticles volume fraction.

Keywords: Hiemenz Flow, Shrinking Sheet, Hybrid Nanofluid

1. Introduction

The Hiemenz flow across a shrinking sheet in hybrid nanofluid has piqued the interest of many researchers. Hiemenz flow is the stagnations point flow by using similarity of the solution to reduce number of variables by means of coordinate transformation. Homann [1] expanded Hiemenz's [2] inquiry into the purpose of the mounted surface's stagnation within the physical phenomenon flow to the situation of the axisymmetric flow. Ariel [3] looked at the hydromagnetic effects on the stream field and came up with a clever solution to the problem. Wang [4] and Waini *et al.* [5-9] have recently considered Hiemenz flow concerns with a variety of implications.

Normal fluid, on the other hand, is used in the cooling systems of most industrial activities. However, Choi and Eastman [11] suggested that a high-level liquid known as nanofluid might improve the liquid's warmth transfer rate. Nanofluid is made up of a single nanoparticle suspended in a liquid basis. Several companies have investigated employing nanofluid in heat transfer upgrades as discovered by [12] and [13]. Turcu *et al.* [14] and Jana *et al.* [15] were the first two studies to use hybrid

*Corresponding author: fazlina@uthm.edu.my

nanoparticles in their experiments. Furthermore, Suresh *et al*. [16] investigated the use of an alumina hybrid nanoparticle to determine whether it may increase fluid heat conduction.

Aside from that, Devi, and Devi [17] investigated the flow over a shrinking surface containing aluminium oxide using magnetohydrodynamics. The mathematical correlations of hybrid nanofluid are provided and tracked down to ensure that the results of Suresh *et al.* [16]'s exhibiting in formation and exploratory information are understood. Then, Ghalambaz *et al.* [18] addressed the flow of a water-based corundum hybrid nanofluid toward a vertical plate for the goal of stagnation. Furthermore, Waini et al. [5-9] looked at the two-fold arrangements of hybrid nanofluid flow, and the issues are currently being investigated under various physical conditions.

For additional research, the literature offers a review of nanofluid and hybrid nanofluid. The objective of this research is to look into Hiemenz flow in a hybrid nanofluid over a shrinking sheet. The mathematical formulation and an attempt to solve the problem using the shooting technique in Maple software with the Runge-Kutta-Fehlberg (RKF45) method. Furthermore, the current numerical results are validated by comparing them to prior research [10]. Aside from the existing findings of Waini *et al.* [10], new findings have been made.

2. Methodology

In this section, the governing equation for Hiemenz flow over a shrinking sheet in hybrid nanofluid problem were discussed. The Runge-Kutta-Fehlberg (RKF45) and shooting method have been applied to solve the problem.

2.1 Mathematical Formulation

The Hiemenz flow of a hybrid nanofluid on a shrinking sheet is considered. The flow configuration of the problem is illustrated in Figure 2.1. Here, the free stream velocity is taken as $u_e(x) = U_e x$, while the surface velocity is $u_w(x) = U_w x$ with U_e and U_w are constants. The governing equations are:

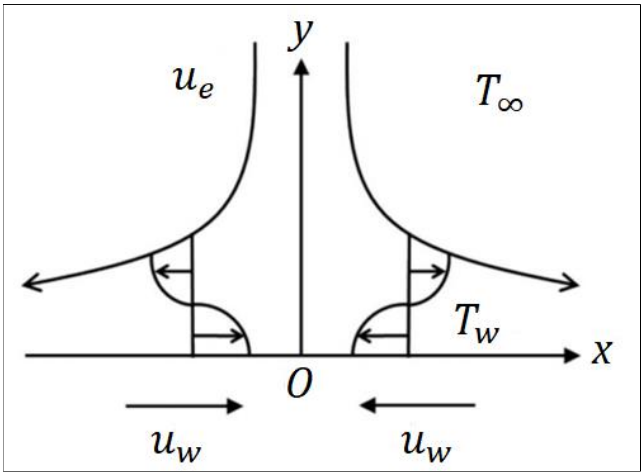

Figure 2.1: The flow configuration

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad Eq.\ 1$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = u_e\frac{du_e}{dx} + \frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2} \qquad Eq.\ 2$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{knf}}{(\rho C_p)_{hnf}}\frac{\partial^2 T}{\partial y^2} \qquad Eq.3$$

subject to:

$$v=0,\quad u=u_w(x),\quad T=T_w$$
$$u\to u_e,\quad T\to T_\infty \text{ as } y\to\infty \qquad Eq.4$$

Where u and v are the velocity components of the hybrid nanofluid along x and y axis, and T is the hybrid nanofluid temperature. Further, for ρ_{hnf}, μ_{hnf}, k_{knf} and $(\rho C_p)_{hnf}$ are the density, dynamic viscosity, thermal conductivity and heat capacity of the hybrid nanofluid, respectively. Also, the fluid, nanofluid and hybrid nanofluid is represent as f, nf, hnf, respectively.

2.2 Similarity Transformation

The similarity variables are as follows:

$$\psi=(U_e v_f)^{\frac{1}{2}}xf(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad \eta=(U_e/v_f)^{\frac{1}{2}}y \qquad Eq.\ 5$$

v_f is represent as the fluid kinematic viscosity while ψ is the stream function which is define as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ which is identically satisfies Eq. 1 using Eq. 5.

$$u=U_e xf(\eta),\quad v=-\left(U_e v_f\right)^{1/2} f(\eta) \qquad Eq.\ 6$$

where the velocity components along the $x-$ and $y-$axes are represented by u and v, and the temperature is given by T. Further, Table 2.1 presents the thermophysical correlations of nanofluid [21] and hybrid nanofluid [17]. Meanwhile, Table 2.2 displays the properties of nanoparticles and water [21]. Note that alumina oxide can copper are the nanoparticles, and their volume fractions are symbolized by φ_1 and φ_2, respectively.

2.3 Governing Ordinary Differential Equations

The ordinary differential equations are

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\frac{1}{v_f}f'''+1-f'^2+ff''=0 \qquad Eq.7$$

$$\frac{1}{\Pr}\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}\theta''+f\theta'=0 \qquad Eq.\ 8$$

subject to the boundary condition,

$$f(0)=0,\qquad f'(0)=\lambda,\qquad \theta(0)=1,$$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty \qquad Eq.\ 9$$

where the Prandtl number $\Pr$ and the shrinking parameter λ are defined as [20]

$$\Pr = \frac{(\mu C_p)_f}{k_f}, \quad \lambda = \frac{U_w}{U_e} \qquad Eq.\ 10$$

with $\lambda > 0$ and $\lambda < 0$ indicate the stretching and shrinking cases, respectively, while $\lambda = 0$ indicates the rigid surface. Note that for $\varphi_1 = \varphi_2 = 0$ (regular fluid) and $\lambda = 0$, Eq. 7 reduces to those of the classical Hiemenz problem [10].

The physical quantities of interest are the skin friction coefficients C_f and the local Nusselt number Nu_x which are defined as

$$C_f = \frac{\mu_{hnf}}{\rho_f u_e^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu_x = -\frac{x k_{hnf}}{k_f (T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad Eq.\ 11$$

where the surface shear stress τ_w and the surface heat flux q_w are respectively given by

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad Eq.\ 12$$

Using Eq. 5, Eq. 11, and Eq. 12, we get

$$\mathrm{Re}_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \qquad \mathrm{Re}_x^{1/2} Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0) \qquad Eq.\ 13$$

where the local Reynolds number is $\mathrm{Re}_x = u_e x / \nu_f$.

Table 2.1: Thermophysical properties of nanofluid and hybrid nanofluid

Properties	Nanofluid	Hybrid Nanofluid
Dynamic viscosity	$\mu_{nf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}}$	$\mu_{hnf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Density	$\rho_{nf} = (1-\varphi_1)\rho_f + \varphi_1\rho_{p1}$	$\rho_{hnf} = (1-\varphi_1)[(1-\varphi_1)\rho_f + \varphi_1\rho_{p1}]$ $+\varphi_2\rho_{p2}$
Heat capacity	$(\rho C_p)_{nf} = (1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{p1}$	$(\rho C_p)_{hnf} = (1-\varphi_2)[(1-\varphi_1)(\rho C_p)_f$ $+\varphi_1(\rho C_p)_{p1}] + \varphi_2(\rho C_p)_{p2}$

Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_{p1} + 2k_f - 2\varphi_1(k_f - k_{p1})}{k_{p1} + 2k_f + \varphi_1(k_f - k_{p1})}$	$\frac{k_{hnf}}{k_{nf}} = \frac{k_{p2} + 2k_{nf} - 2\varphi_2(k_{nf} - k_{p2})}{k_{p2} + 2k_{nf} + \varphi_2(k_{nf} - k_{p2})}$ where $\frac{k_{nf}}{k_f} = \frac{k_{p1} + 2k_f - 2\varphi_1(k_f - k_{p1})}{k_{p1} + 2k_f + \varphi_1(k_f - k_{p1})}$

Table 2.2: Thermophysical properties of nanoparticles and water

Properties	Al_2O_3	Cu	Water
$C_p (J/kgK)$	765	385	4179
$\rho(kg/m^3)$	3970	8933	997.1
$k(W/mK)$	40	400	0.613
Prandtl number, $\Pr$			6.2

3. Results and Discussion

In this chapter, the solution is obtained by applying the shooting technique with Runge-Kutta-Fehlberg (RKF45) method in Maple software. Classical Hiemenz problem can be obtained by taking $\varphi_1 = \varphi_2 = 0$ for the regular fluid and $\lambda = 0$ (rigid surface). Therefore, the value obtained is $f''(0) = 1.232588$, which is equivalent to that published by Wang [4], Bachok *et al.* [18] and [10]. The Prandtl number of the base fluid is constant at $\Pr = 6.2$ under various parameters.

The result of this study is compared with previous study performed by Wang [4], Bachok *et al.* [18] and Waini *et al.* [10] for validation. Table 3.1 shows the values of $f''(0)$ with variant of λ for $\varphi_1 = \varphi_2 = 0$. When the $\lambda = 1$ there is no friction occur, so the result is zero. It is observed that the comparison is in good agreement.

Table 3.1: The result of $f''(0)$ with variant of λ for $\varphi_1 = \varphi_2 = 0$

λ	Wang [4]	Bachok *et al.* [18]	Waini *et al.* [10]	Present Result
2	-1.88731	-1.887307	-1.887307	-1.8873067
1	0	0	0	0
0.5	0.71330	0.713295	0.713295	0.713295
0	1.232588	1.232588	1.232588	1.232588
-0.5	1.49567	1.495670	1.495670	1.495670
-1	1.32882	1.328817	1.328817	1.328817
-1.15	1.08223	1.082231	1.082231	1.082231
-1.2		0.932473	0.932473	0.932473
-1.2465	0.55430	0.584281	0.584281	0.584281

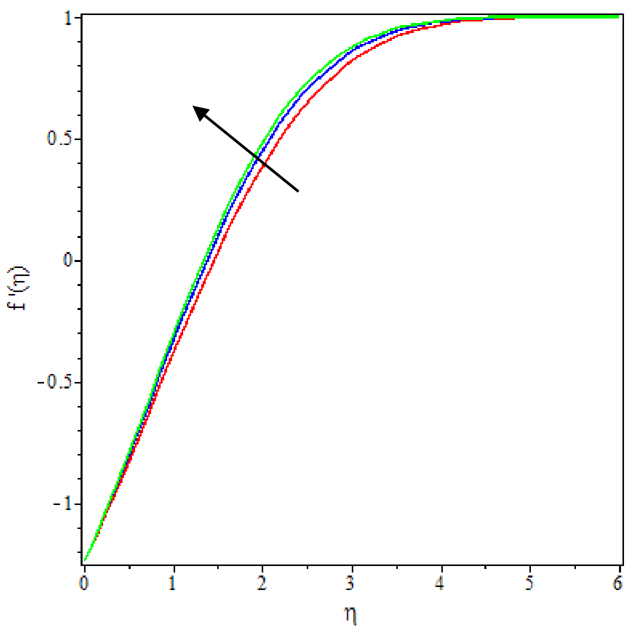

Figure 3.1: Effect of φ_2 on $f'(\eta)$.

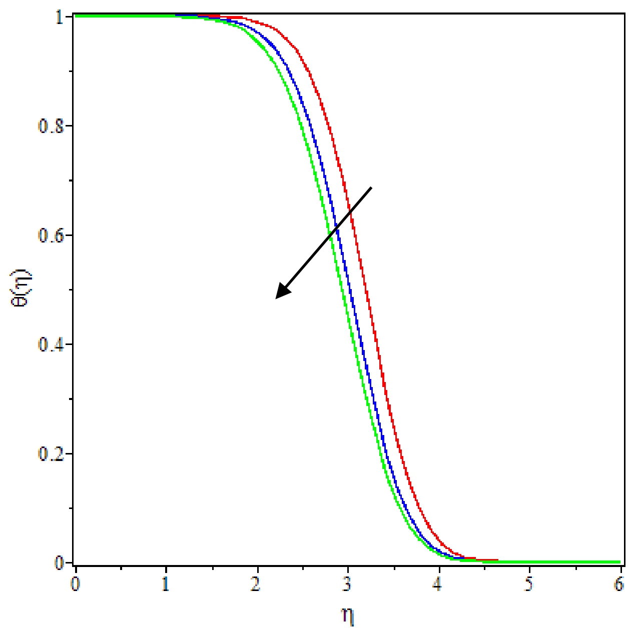

Figure 3.2: Effect of φ_2 on $\theta(\eta)$.

The velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for $\varphi_2 = 0, 0.03, 0.05$ when the $\varphi_1 = 0.05$, $\lambda = -1.24$, and $\Pr = 6.2$ are shown in Figures 4.1 and 4.2, respectively. The $\lambda = -1.24$ meaning that the critical point. The results reveal that as copper nanoparticles volume fraction φ_2 increases, both friction on the surface and the heat transfer rate are also increase for the hybrid nanofluid.

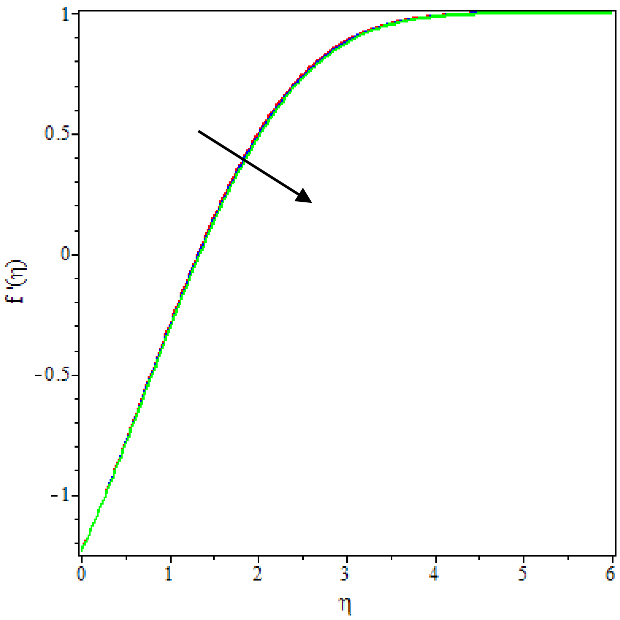

Figure 3.3: Effect of φ_1 on $f'(\eta)$.

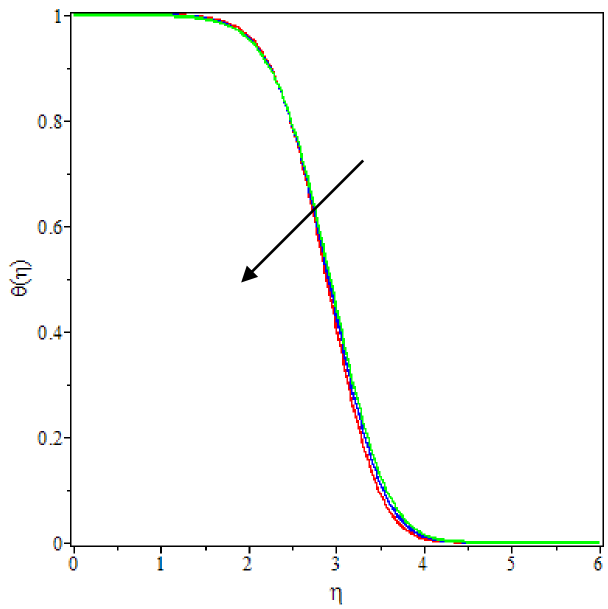

Figure 3.4: Effect of φ_1 on $\theta(\eta)$.

While the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for alumina oxide nanoparticles volume fraction $\varphi_1 = 0, 0.03, 0.05$ when $\varphi_2 = 0.05$, $\lambda = -1.24$, and $\Pr = 6.2$ are shown in Figures 4.3 and 4.4, respectively. The $\varphi_2 = 0.05, \lambda = -1.24$ meaning that the critical point. The results show the opposite behavior than the results of copper nanoparticles volume fraction.

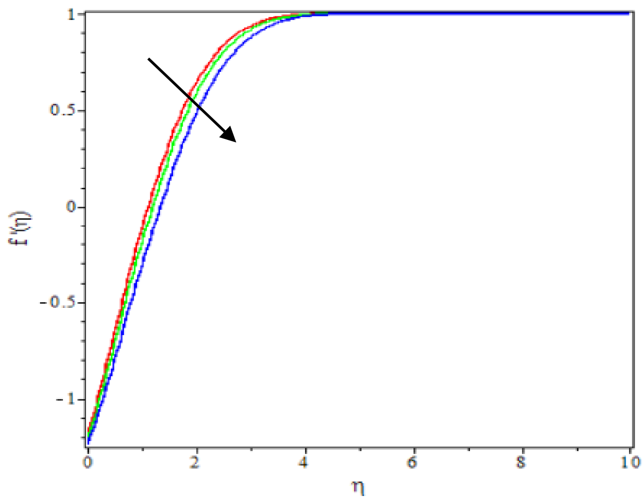

Figure 3.5: Effect of λ on $f'(\eta)$.

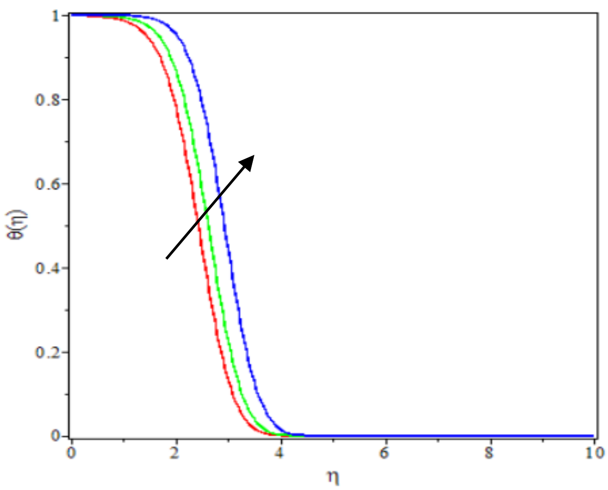

Figure 3.4: Effect of λ on $\theta(\eta)$.

The impact of λ on $f'(\eta)$ and $\theta(\eta)$ is demonstrated in Figures 4.5 and 4.6, respectively, value of λ is the non-uniqueness of the solutions for the shrinking case. It can be seen that the velocity profiles decrease while the temperature profiles increase as the shrinking parameter escalates. Therefore, reduce the skin friction and rise the heat transfer rate at the surface of the hybrid nanofluid.

4. Conclusion

The effect of several physical characteristics on the Hiemenz flow's behavior on a shrinking sheet has been investigated. The governing equation has been transformed from the partial differential equations to ordinary differential equations using similarity transformation before been solved using RKF45 method with shooting technique in Maple software. New results have been found that is the impact of alumina oxide is to decrease the heat transfer rate and the skin friction on the surface for a fixed value of copper nanoparticles volume fraction and for a fixed value of alumina oxide nanoparticles volume fraction, copper has been observed to reduce the heat transfer rate and skin friction on the surface. Besides that, the shrinking parameter reduces the skin friction and rises the heat transfer at the surface of the hybrid nanofluid.

Acknowledgement

The author would like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

References

[1] Der HF. Einflub grober Zähigkeit bei der Strömung um den Zylinder und um die Kugel. Zeitschrift Für Angew Math Und Mech 1936;16:153–64

[2] Hiemenz K. Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszylinder. Dinglers Polytech J 1911;326:321–410

[3] Ariel PD. Hiemenz flow in hydromagnetics. Acta Mech 1994;103:31–43.

[4] Wang CY. Stagnation flow towards a shrinking sheet. Int J Non Linear Mech 2008;43:377–82.

[5] Waini I, Ishak A, Pop I. Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid. Int J Heat Mass Transf 2019;136:288–97.

[6] Waini I, Ishak A, Pop I. MHD flow and heat transfer of a hybrid nanofluid past a permeable stretching/shrinking wedge. Appl Math Mech (English Ed) 2020;41:507–20.

[7] Waini I, Ishak A, Groşan T, Pop I. Mixed convection of a hybrid nanofluid flow along a vertical surface embedded in a porous medium. Int Commun Heat Mass Transf 2020;114. Article ID 104565.

[8] Waini I, Ishak A, Pop I. Hybrid nanofluid flow towards a stagnation point on an exponentially stretching/shrinking vertical sheet with buoyancy effects. Int J Numer Meth Heat Fluid Flow 2020.

[9] Waini I, Ishak A, Pop I. Hybrid nanofluid flow towards a stagnation point on a stretching/shrinking cylinder. Sci Rep 2020;10. Article ID 9296.

[10] Waini, I., Ishak, A., & Pop, I. (2020). Hiemenz flow over a shrinking sheet in a hybrid nanofluid. *Results in Physics*, *19*, 103351.

[11] Choi SUS, Eastman JA. Enhancing thermal conductivity of fluids with nanoparticles. Proc 1995 ASME Int Mech Eng Congr Expo FED 231/MD 1995;66:99–105.

[12] Hassan M, Faisal A, Ali I, Bhatti MM, Yousaf M. Effects of Cu–Ag hybrid nanoparticles on the momentum and thermal boundary layer flow over the wedge. Proc Inst Mech Eng Part E J Process Mech Eng 2019;233:1128–36.

[13] Mahanthesh B, Shehzad SA, Ambreen T, Khan SU. Significance of Joule heating and viscous heating on heat transport of MoS2–Ag hybrid nanofluid past an isothermal wedge. J Therm Anal Calorim 2020.

[14] Turcu R, Darabont A, Nan A, Aldea N, Macovei D, Bica D, et al. New polypyrrolemultiwall carbon nanotubes hybrid materials. J Optoelectron Adv Mater 2006;8:643–7.

[15] Jana S, Salehi-Khojin A, Zhong WH. Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives. Thermochim Acta 2007;462:45–55.

[16] Suresh S, Venkitaraj KP, Selvakumar P, Chandrasekar M. Synthesis of Al2O3-Cu/ water hybrid nanofluids using two step method and its thermo physical properties. Colloids Surfaces A Physicochem Eng Asp 2011;388:41–8.

[17] Devi SPA, Devi SSU. Numerical investigation of hydromagnetic hybrid Cu- Al2O3/ water nanofluid flow over a permeable stretching sheet with suction. Int J Nonlinear Sci Numer Simul 2016;17:249–57.

[18] Ghalambaz M, Rosca NC, Rosca AV, Pop I. Mixed convection and stability analysis of stagnation-point boundary layer flow and heat transfer of hybrid nanofluids over a vertical plate. Int J Numer Meth Heat Fluid Flow 2020;30:3737–54.

[19] Bachok N, Ishak A, Pop I. Stagnation-point flow over a stretching/shrinking sheet in a nanofluid. Nanoscale Res Lett 2011;6. Article ID 623.

[20] Schlichting H, Gersten K. Boundary Layer Theory. 8th ed. Berlin: Springer; 2003.

[21] Oztop HF, Abu-Nada E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. Int J Heat Fluid Flow 2008;29:1326–36.

[22] Schlichting H, Gersten K. Boundary Layer Theory. 8th ed. Berlin: Springer; 2003.