Enhanced Knowledge in Sciences and Technology Vol. 2 No. 1 (2022) 423-432

EKST

Homepage: http://publisher.uthm.edu.my/periodicals/index.php/ekst
e-ISSN : 2773-6385

Application of SIR Model to Analyze the Spreading of Covid-19

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DOI: https://doi.org/10.30880/ekst.2022.02.01.046
Received 02 January 2022; Accepted 02 March 2022; Available online 1 August 2022

Abstract: Covid-19 disease situation has been worrying people around the world because it has caused many deaths and spread easily through the respiratory droplets. This research aims to analyze the stability of susceptible-infected-recovered (SIR) model for spreading of Covid-19 using Maple and to perform the simulation of SIR model using Excel besides comparing the data of Covid-19 between 2020 and 2021 in Malaysia. This study focused on analyzing the spreading of Covid-19 in Malaysia by applying the SIR model to real data. The data spreading of Covid-19 in Malaysia have been used in this study were in the interval between 18 March 2020 until 31 October 2021. In addition, in this research, the stability analysis was carried out with two equilibrium points which include the disease-free equilibrium point and endemic equilibrium point using Maple software. The dynamics behavior of Covid-19 epidemic is verified by generating a simulation of phase portrait for susceptible (S) versus infected (I) where the results showed that the spreading increased in certain time because of the phase portrait of SIR model is semi stable. Lastly, the SIR model was verified in applying the model to real data of spreading Covid-19 to analyze its trend and compared the data between 2020 and 2021 graphically.

Keywords: Covid-19, SIR Model, Stability Analysis, Disease-Free Equilibrium Point, Endemic Equilibrium Point

1. Introduction

Coronavirus disease 2019 (COVID-19) can be defined as an illness caused by a novel coronavirus which is called the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2; formerly called as 2019-nCoV). Based on the situation report by [1], worldwide, there were more than 3,759,967 cases of infected people while about 259,474 cases of death due to COVID-19 virus infection between December 2019 and 8 May 2020. The scientists have been found the specific vaccinations that can cure the disease successfully but, the dynamics of spreading of Covid-19 should be investigated to study the behaviour of the virus to the people. Thus, the epidemiologists and mathematicians have identified an

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effective and valuable methods that can be used for comprehending the dynamics in the infectious disease [2].

The Kermack-McKendrick epidemic model, considered as one of the first compartmental models, was established in the late 1920s comprises a system of three ordinary differential equations [3]. According to [4], the SIR model helps to provide the observations and forecasting on the transmission of the disease in communities. Besides that, SIR model is easier to apply for analyzing the spreading of Covid-19 compared to SEIR model which it is complex and necessitates the use of assumptions prior for modelling. As a result, SIR model is intended to simplify many complexities connected with real-time development of the propagation of virus in a way which is both quantitative and qualitatively meaningful.

The aim of this research is to analyze the stability of SIR model for spreading of Covid-19 using Maple. The following objective is to perform simulation of SIR model for Covid-19 disease using Excel. Lastly, the comparison of Covid-19 data in Malaysia between 2020 until 2021 have been carried out graphically.

2. Methodology

2.1 Formulation of SIR Model

Based on Kermack and McKendrick (1927) model, these are the following set of differential equations of classical Susceptible, Infected and Recovered (SIR) epidemic dynamics model that has been used in this study is written as [5],

$$\frac{dS}{dt} = -\frac{\beta SI}{N} \qquad \textit{Eq. 1}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I \qquad \textit{Eq. 2}$$

$$\frac{dR}{dt} = \gamma I \qquad \textit{Eq. 3}$$

where

- N = total population
- S = number of susceptible individuals
- I = number of infected individuals
- R = number of recovered individuals
- β = effective amount of people infected per day
- γ = the proportion of infected recovering per day ($\gamma = \frac{1}{D}$)
- D = number of days an infected person has and can spread the disease
- $R_0 = \frac{\beta}{\gamma}$ is the basic reproduction number

2.2 Stability Analysis of SIR Model

The stability analysis is used to analyze the spreading of Covid-19 with two equilibrium points which includes the disease-free equilibrium point and endemic equilibrium point [6].

a) Stability analysis at the disease-free equilibrium point.
Solving *Eq. 1*, *Eq. 2* and *Eq. 3*, we have conditions,

$$I(t) = 0 \qquad \textit{Eq. 4}$$

$$0 = -\frac{\beta SI}{N} \qquad Eq.\ 5$$

$$S(t) = 0 \qquad Eq.\ 6$$

$E_0 = (0,0,0)$ is the disease-free equilibrium point.

By using Maple software, the analysis started with *Eq. 1* and *Eq. 2* :

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$

Based on the above equations, found the Jacobian matrix,

$$J = \begin{bmatrix} -\frac{\beta I}{N} & -\frac{\beta S}{N} \\ \frac{\beta I}{N} & \frac{\beta S}{N} - \gamma \end{bmatrix} \qquad Eq.\ 7$$

where the value of basic reproduction number is [5],

$$R_0 = \frac{\beta}{\gamma}\ , \quad \beta = R_0\gamma$$

the β value has substituted into the Jacobian matrix,

$$J = \begin{bmatrix} -\frac{R_0\gamma I}{N} & -\frac{R_0\gamma S}{N} \\ \frac{R_0\gamma I}{N} & \frac{R_0\gamma S}{N} - \gamma \end{bmatrix} \qquad Eq.\ 8$$

$$J = \begin{bmatrix} 0 & 0 \\ 0 & -\gamma \end{bmatrix} \qquad Eq.\ 9$$

the determinant for the matric is,

$$det \begin{bmatrix} -\lambda & 0 \\ 0 & -\gamma - \lambda \end{bmatrix} \qquad Eq.\ 10$$

$$\lambda(\gamma + \lambda) = 0 \qquad Eq.\ 11$$

which has gives the eigenvalues,

$$\lambda_1 = 0\ , \quad \lambda_2 = -\gamma \qquad Eq.\ 12$$

Based on the behavior of disease-free equilibrium point which is non-hyperbolic, because there is a root of the equation with $\lambda_1 = 0\ ,\ \lambda_2 = -\gamma$.

b) Stability analysis at endemic equilibrium point.
From *Eq.2* at time, *t*, $I(t)$ with a condition that $I(t) \neq 0$.

$$0 = \frac{\beta SI}{N} - \gamma I \qquad Eq.\ 13$$

$$0 = I(\frac{\beta SI}{N} - \gamma) \qquad Eq.\ 14$$

$$S(t) = \frac{N\gamma}{\beta} \qquad \textit{Eq. 15}$$

$E_1 = (\frac{N\gamma}{\beta}, 0,0)$ which is the endemic equilibrium point at $I(t) \neq 0$.

By using Maple software, the analysis has proceeded with *Eq. 1* and *Eq. 2,*

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$

the above equation has given the Jacobian matrix as,

$$J = \begin{bmatrix} -\frac{\beta I}{N} & -\frac{\beta S}{N} \\ \frac{\beta I}{N} & \frac{\beta S}{N} - \gamma \end{bmatrix} \qquad \textit{Eq. 16}$$

the point of $E_1 = (\frac{N\gamma}{\beta}, 0,0)$ substituted to the Jacobian matrix,

$$J = \begin{bmatrix} 0 & -\beta(\frac{N\gamma}{\beta}) \\ 0 & \beta(\frac{N\gamma}{\beta}) - \gamma \end{bmatrix} \qquad \textit{Eq. 17}$$

$$J = \begin{bmatrix} 0 & -\gamma \\ 0 & 0 \end{bmatrix} \qquad \textit{Eq. 18}$$

the determinant for the matric is,

$$det \begin{bmatrix} -\lambda & -\gamma \\ 0 & 0 - \lambda \end{bmatrix} \qquad \textit{Eq. 19}$$

$$\lambda^2 = 0 \qquad \textit{Eq. 20}$$

and found the eigenvalues,

$$\lambda_{1,2} = 0 \qquad \textit{Eq. 21}$$

Hence, the equations of the spread of disease at equilibrium points is a fixed equilibrium point.

2.3 Data

In this study, the data spreading of Covid-19 in Malaysia have been used for simulation of SIR model. The data was obtained from [7] and through the KKM website [8,9]. In addition, the simulation of SIR model has been carried out with real data of Covid-19 outbreak in the interval between 18 March 2020 until 31 October 2021.

3. Results and Discussions

3.1 Stability Analysis of SIR Model of Covid-19

The stability analysis was used to understand the severity of spreading of Covid-19 in Malaysia with the disease-free and endemic equilibrium point by presenting the phase portrait of dynamical system. Figure 1 shows the phase portrait of susceptible (S) versus infected (I) by Covid-19 with the parameters value that have been assumed for $\beta = 0.5$ and $\gamma = 0.3$. The parameters value that has been assumed was obtained from [6].

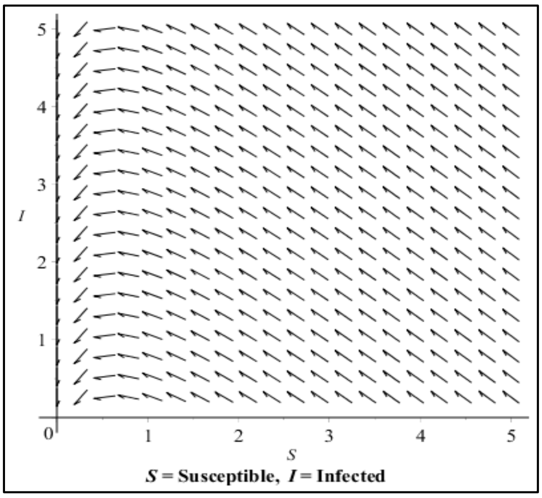


Figure 1 : Phase portrait of Susceptible (*S*) versus Infected (*I*) with assuming $\beta = 0.5$ and $\gamma = 0.3$.

Based on the phase portrait results, the equilibrium of SIR model at the equilibrium point, $E_0 = (0,0,0)$, there are two conditions of eigenvalues, $\lambda_1 = 0$ and $\lambda_2 = -\gamma$. The conditions are if $\gamma > 0$, the eigenvalue will be negative while if $\gamma < 0$, the eigenvalue will be positive.

From the results, when eigenvalue of $\lambda_1 = 0$, the stability of directional field moves away from the line along the equilibrium point, *S*. Thus, the state of equilibrium point in this area will be unstable. Meanwhile, when the parameter of eigenvalue is $\lambda_2 = -\gamma$ where $\gamma > 0$, the stability of directional field moves slowly towards the disease-free equilibrium point, $E_0 = (0,0,0)$. The black line or the stability of directional field shows a slope which moves closer to the equilibrium point so that the state of point is stable and it illustrates that the Covid-19 disease is spreading widely.

Furthermore, the stability for $\lambda_{1,2} = 0$ will be constant at the endemic equilibrium point, $E_1 = (\frac{N\gamma}{\beta}, 0,0)$. According to [10], if both eigenvalues are negative, the disease-free and endemic equilibrium point are locally asymptotically stable. Therefore, the phase portrait of SIR model is semi stable and this showed that the dynamics of Covid-19 disease will increase in certain time [6].

3.2 Real Data of Spreading of Covid-19 in Malaysia for Year 2020 and 2021

The real data of spreading of Covid-19 in Malaysia which began on 18 March 2020 and ended on 31 October 2021 have been considered to use in this study for comparison. The graphs of daily and total infections were presented to compare their trends between year 2020 and 2021. Figure 2 shows that the number of infections on 18 March 2020 was 117 cases [8]. The number of cases had a sudden rise to 691 on 6 October 2020 compared to the previous day. Figure 3 illustrates the total number of infections during the same interval and reached 113 010 cases on 31 December 2020 [8].

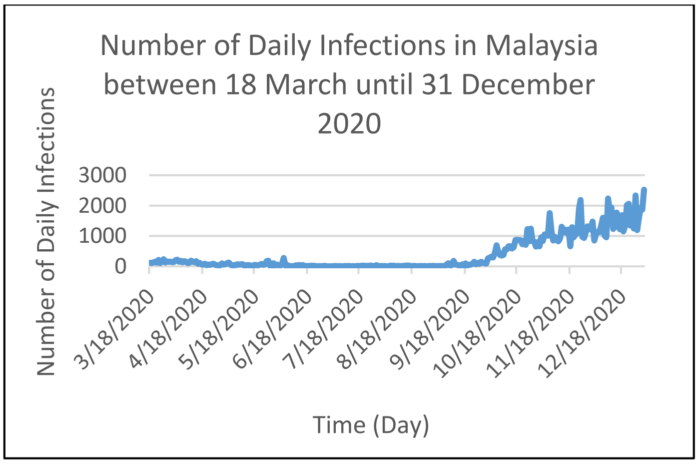


Figure 2 : Number of Daily Infections in Malaysia between 18 March until 31 December 2020, Source: Ministry of Health (2020).

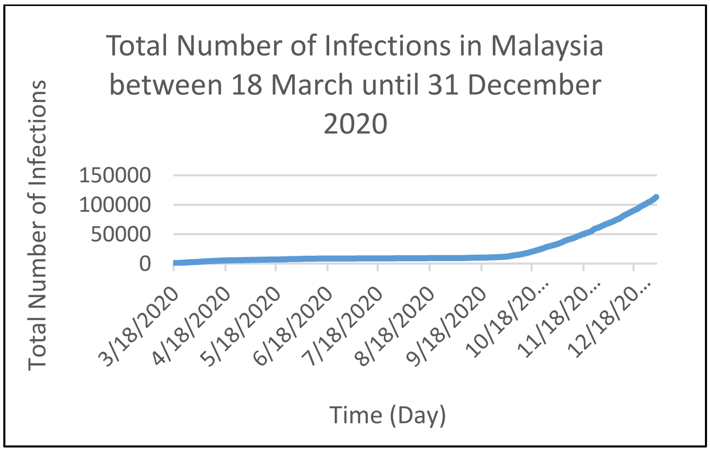


Figure 3 : Total Number of Infections in Malaysia between 18 March until 31 December 2020, Source: Ministry of Health (2020).

Figure 4 represents the number of infections of Covid-19 disease for year 2021 began with 2068 cases on 1 January 2021 and then rise substantially until reached a climax with 24 599 new cases on 26 August 2021 [9]. Next, Figure 5 shows the total number of infections during the same period began which ended with 2 471 642 cases on 31 October 2021 [9].

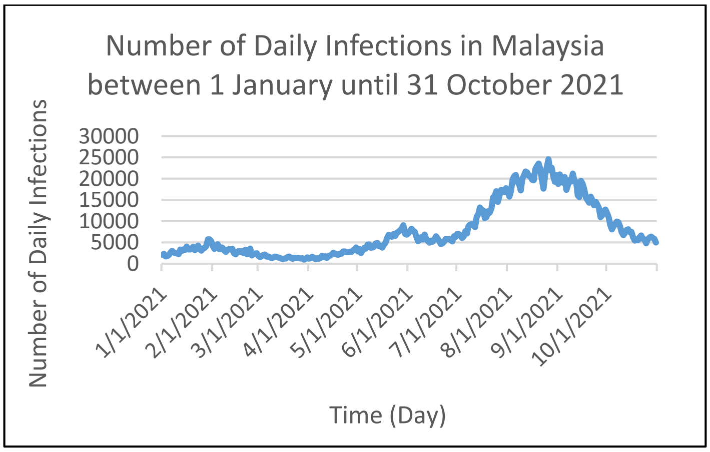


Figure 4 : Number of Daily Infections in Malaysia between 1 January until 31 October 2021, Source: Ministry of Health (2021).

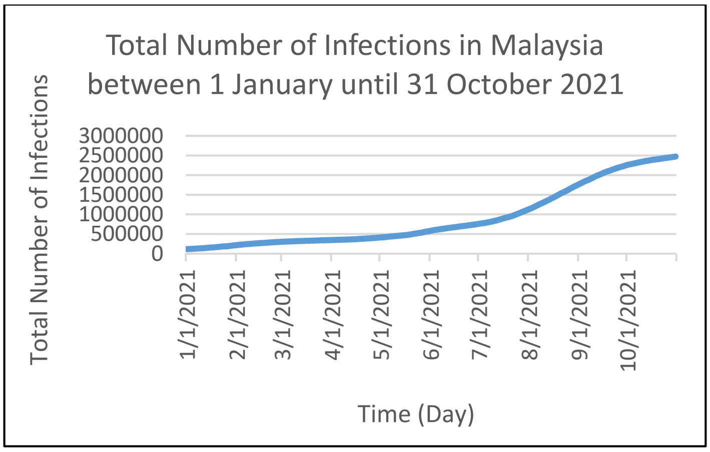


Figure 5 : Total Number of Infections in Malaysia between 1 January until 31 October 2021, Source: Ministry of Health (2021).

Based on the curves of daily infections and the total infections of Covid-19 in Malaysia between year 2020 and 2021, there are significant changes on the shape of both graphs. The number of infections of Covid-19 in Malaysia for year 2021 was higher than 2020 based on the information of health in KKM website.

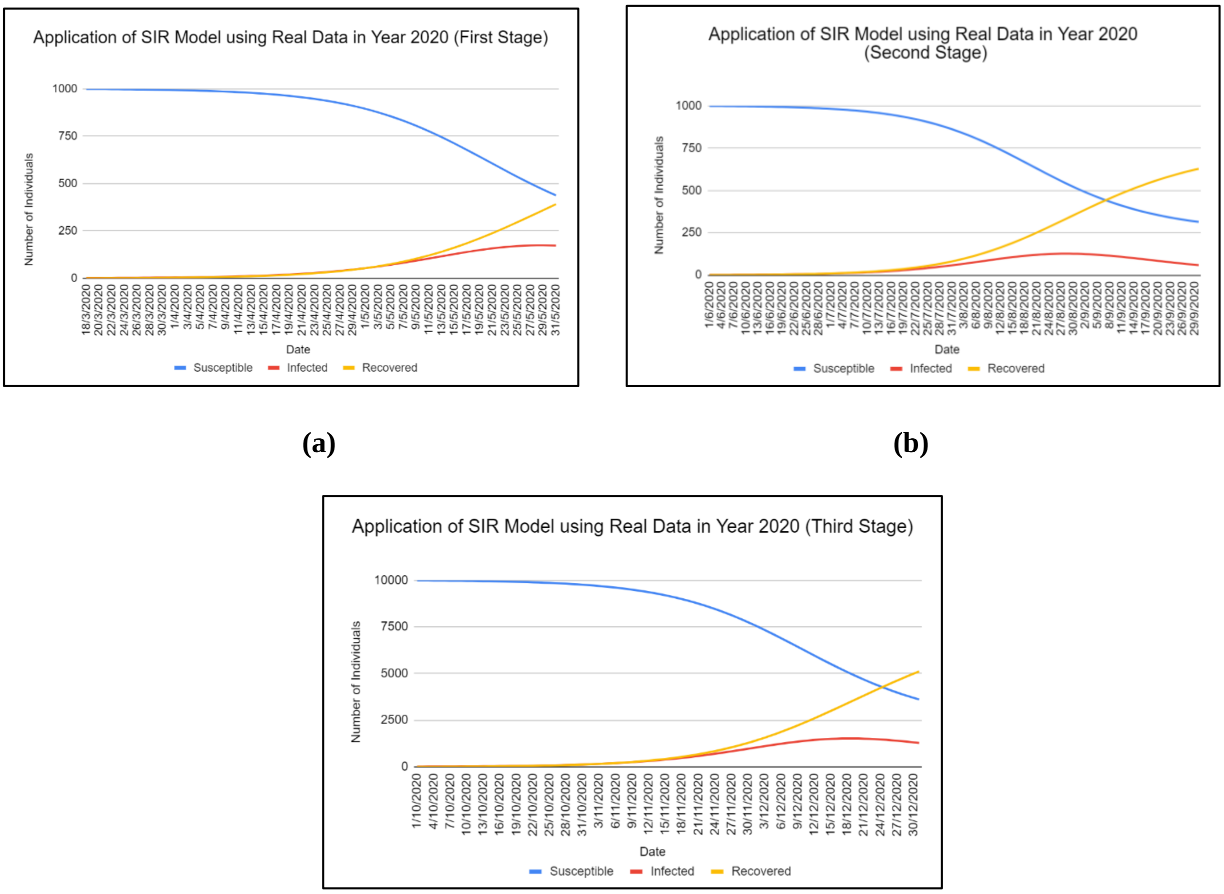
3.3 Application of SIR Model Using Real Data of Covid-19 in Malaysia

The simulation of SIR model has divided into two stages which the first stage consisted of real data Covid-19 in year 2020 while second stage using the data in year 2021. We divided into several stages based on the value of β. The official data according to the Department of Statistics Malaysia (DOSM) have been used to calculate the total population in this country where the value of S (0) = 32 670 000 [11]. There were consisted by three stages for year 2020 while two stages for 2021 where this analysis has been done with the simulation of SIR model using the Excel. Table 1 represents the values of parameters which have been used to build the SIR model graph in year 2020 and 2021. These values were calculated from *Eq. 1*, *Eq. 2* and *Eq. 3.*

Table 1 : Values of Parameters for Year 2020 and 2021

Parameter	Value of Parameters for year 2020			Value of Parameters for year 2021	
	First stage	Second stage	Third stage	First stage	Second stage
R_0, (β/γ)	2.09	1.83	1.97	1.78	1.60
β	0.190	0.170	0.179	0.162	0.145
D	11	11	11	11	11
γ, $(1/D)$	0.09	0.09	0.09	0.09	0.09

Figure 6 show the graphs of SIR model that applied to real data of spreading Covid-19 for first, second and third stage in year 2020. The analysis of SIR model used different value of effective amount of people infects per day, β which has obtained from the reproduction number and the proportion of infected recovering per day, γ. From the results of three graphs, the trend of susceptible were same which consistent initially before dropped slowly. Based on the Figure 6a, the first stage was applied by real data of Covid-19 spread in the interval between 18 March until 31 May 2020 and used value, β = 0.190. The number of infected has been increased on 3 May 2020 and its trend was continues to arise until 31 May 2020 while the simulated recovered trend begins to rise after 3 May 2020. Meanwhile, Figure 6b represents the second stage were started on 1 June until 30 September 2020 and have used value, β = 0.170. The number of infected were plateaued around 30 August 2020 and steadily declined until 30 September 2020 while the number of recovered has risen significantly until at the end of September 2020. Besides that, Figure 6c illustrates the third stage in 2020 were applied by real data in the interval between 1 October until 31 December 2020 using value, β = 0.179. The number of infected were increased gradually around 24 November 2020 before its trend dropped slowly until 31 December 2020. At the same time, the number of recovered rises rapidly until at the end of December 2020.

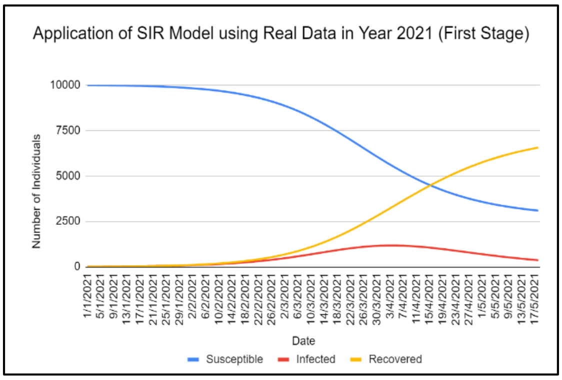


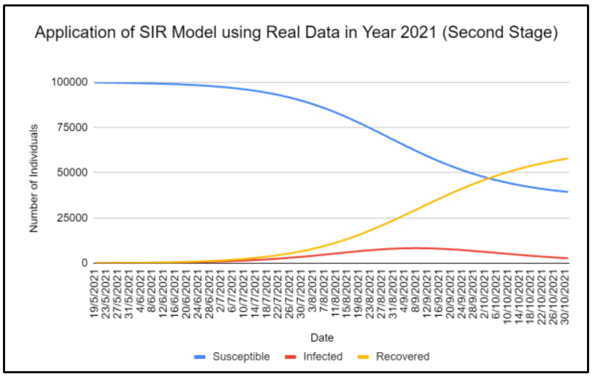
(a) **(b)** **(c)**

Figure 6 : Application of SIR Model using Real Data in Year 2020. (a) First Stage (β = 0.190), (b) Second Stage (β = 0.170), (c) Third Stage (β = 0.179).

In addition, Figure 7 represents the graphs of SIR model that applied to real data of spreading Covid-19 for first and second stage in year 2021. The real data that applied to SIR model for first stage was between 1 January until 18 May 2021 while the second stage was applied by real data in the interval between 19 May until 31 October 2021. Figure 7a has used value, β = 0.162 that evaluated from the reproduction number in the data provided. At first, both graphs in Figure 7 show that the number of susceptible were consistent but, the trends become decreased gradually until at the end of the time period. Then, the number of infected would have higher peaked than in second stage for year 2021 before its trend steadily declined until the end of interval. The number of recovered has risen and its simulated trend continues to rose significantly until 18 May 2021.

Moreover, the Figure 7b has used value, β = 0.145 to obtain the pattern of trends in susceptible, infected and recovered on the second stage. The graph shows that the number of infected were a little bit of plateaued compared to first stage which around 31 August 2021 but, its trend has decreased gradually until 31 October 2021. Lastly, the number of recovered has increased on 12 September and continues to rise substantially until 31 October 2021.





(a) **(b)**

Figure 7 : Application of SIR Model using Real Data in Year 2021. (a) First Stage (β = 0.162), (b) Second Stage (β = 0.145).

3.4 Summary

Based on the results of application SIR model to real data of Covid-19 spread, the trend of the graphs for every stage in year 2020 and 2021 are same and unchanged although the number of individuals are different. The trends of susceptible people were decreases substantially as the trends of infected people increases significantly which happened in every stage for both years. The peak of infections was different for each stage because of the vaccination percentage rate which have increased during 2021. Meanwhile, the trends of recovered people were rise gradually due to immunization for every stage in year 2020 and 2021.

4. Conclusion

In this study, the dynamics of spreading of Covid-19 in Malaysia can be analyzed with more details using the SIR model because it is an effective approach for measuring and controlling the Covid-19 epidemic. The stability analysis has carried out with two equilibrium points which were the disease-free and endemic equilibrium point using the Maple software. From the result, the dynamic behaviour of Covid-19 epidemic at susceptible (S) and infected (I) was obtained as semi stable which means that the Covid-19 disease will be increased over the certain time [6]. Moreover, the real data of spreading Covid-19 disease started on 18 March 2020 until 31 October 2021 have compared graphically which includes the number of daily and total number infections in Malaysia. The results obtained shows that the number of infections of Covid-19 disease in year 2021 was higher than 2020. Next, the real data of spreading Covid-19 in Malaysia have been used to simulate the results from SIR model using Excel. The simulation of SIR model to real data have been successful and found that the number of infections in 2020 have a higher peak than 2021 due to a high percentage rate of vaccination.

This study can be further with some recommendations to analyze the spreading of Covid-19 with more details. Firstly, the SIR model can be added with more parameters such as rate of births and rate of natural death. Besides that, this study can be further by modifying the SIR model to become any modified mathematical models such as SEIR model with adding the exposed group. In addition, the application of SIR model to the real data can be simulated using many mathematical softwares such as Matlab and R-Python. Lastly, this study can be further by comparing the data of Covid-19 outbreak in Malaysia with other countries.

Acknowledgement

This research was made possible by funding from research grant number H962 provided by the Ministry of Higher Education, Malaysia. The authors would also like to thank the Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia for its support.

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