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Identify Factors to Minimize Ruin Probability in Insurance Industry

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Abstract: The level of an insurer's excess for a portfolio of insurance policies is the subject of ruin theory. The purpose of this research is to investigate the factors to minimize the probability of ruin in insurance industry by reinsurance. Initially the total loss on a unit interval has a light-tailed distribution or known as exponential distribution, and a heavy-tailed distribution or known as Pareto distribution. Then, study the process of reinsurance in the insurance environment. The problem is solved using Microsoft Excel.

Keywords: Ruin Probability, Reinsurance, Hamilton Jacobi Bellman Equation

1. Introduction

[1] According to J. Francois Outreville, the fundamental requirement for the existence of insurance contracts is the existence of a large number of similar loss exposures. Ruin probability is an event faced by the insurance company which insurance claiming by client were exceed the total premium collected and the sum of initial surplus. Reinsurance is one of the initiative to minimize ruin probability and quite popular among insurance company. Reinsurance is an action taken by insurance company which transfer parts of risk portfolio to different parties with some form of agreement to decrease the likelihood of paying large obligation caused by insurance claims. Reinsurance act as a solvent by allowing insurers to reclaim some or total amount that have been claim by insureds. Generally, the insurance company that apply reinsurance is transferring the risk to another company in order to minimize the likelihood of huge claim event happen. There are 2 types of reinsurance contract, proportional, and non-proportional. In proportional reinsurance, all policy premium that received by the insurance company, need to be share with reinsurance company. As an exchange, the reinsurer will face some losses based on the negotiation percentage. The reinsurer also need to pay the insurer the cost of processing, business acquisition, and writing. While non-proportional reinsurance or also known as excess of loss reinsurance. The reinsurer will be paying excess losses up to a maximum limit of the ceding company's retention limit. Premium of the reinsurance is calculated independently of the premium charged to the insured.

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2. Materials and Methods

2.1 Insurance Model

In practice, insurance companies face different crisis, the most common are the assessment, that is, the calculation of a premium in the risk capital assessment, the assessment in the calculation of the deductible, the behaviour of the company's management in relation to the nature of the risk. Each subject is very important to the business of an insurance company. It is in the company's interest to charge a higher insurance premium to avoid making a loss. In this case, however, most policyholders would go to another insurance company with cheaper insurance.

The issue of reinsurance is the pricing and retention of risk capital, and the maximum tolerable level of risk that a company can cover with that capital. All insurers have a certain amount of capital used to resolve customer claims. If an entity insures a high-risk event, that event may exceed the entity's total capital available. In this case, the business shares risky reservations between reinsurers and maintains the portion covered by the available capital. In order not to lose, the insurer will decide exactly how much venture capital he wants to maintain for himself.

From a theoretical factor of view, a mathematical version may be referred to as a reservoir. The function of this version is that on the only hand, capital inflows are regular, while on the opposite hand, capital outflows are indeterminate. Capital outflows may be very abnormal relying on unexpected occasions including injuries and herbal disasters. We can see that the random nature of the capital outflow is double that, that means we don`t recognise whilst it is going to be claim or the dimensions of the claim. This easier version describes the maximum crucial elements of coverage problems.

Mathematical model of insurance depends 2 stochastic and other 2 deterministic elements. The elements include:

1. The beginning of the process defined by u or also known as initial reserve;
2. Profit determined by premium return c;
3. Time sequence when it comes to the first claim $T_1, T_2, T_3, \ldots$, where T_1 refer to the time interval between the beginning moments $t = 0$ and the moment when it comes to the first claim.
4. The claim variable sequence $X_1, X_2, X_3, \ldots$ determining decreases of the capital.

Note that the sequence (T_i) and (X_i) are independent sequence of random values.

In addition to above values, the following values are also important in insurance theory:

1. $N(t)$ or numerical process represent the number of claim up to time t and get

$$N(t) = \sup\{n \geq 1 | T_n \leq t\}, t > 0 \qquad \textit{Eq. 1}$$

2. Following expression defined amount of claim up totalled to time t:

$$S(t) = \sum_{i=1}^{N(t)} X_i \qquad \textit{Eq. 2}$$

When insurance company's profit, $U(t)$, then following formula could be obtained:

$$U(t) = u + ct - S(t) \qquad \textit{Eq. 3}$$

Variable S and N is very crucial in risk. When both value were obtained, it would lead to amount of risk to be able to calculate. These two series of basic variable could help to derives all of other variable. This means that the damage distribution X_i ought to be describe within side the identical manner of time T_i after damage were paid.

2.2 Reinsurance Model

In the case of quota share, the capital insurer allocates a certain percentage of each risk covered by the capital insurer within the business unit covered by the contract. The reinsurer participates in the written premium (minus the placement fee) and pays equal amount of each damage.

Quota stocks are widely used in liability insurance contracts (other than auto insurance) because they are easy to manage and do not favour reinsurers. These types of contracts are usually in favor of reinsurers, so higher fees and better terms can be achieved. The quota share is the most efficient contract for SMEs to enter new industries and reduce 4,444 unaccounted for insurance premiums. Quota sharing is also ideal for mutual agreements between insurance companies. For example, two insurers with equal volume and profitability may reinsurance 50% of each other's business. This can have a significant diversification effect on both sides, especially if you do business in geographically different regions.

For the formulation of the model, classical risk of diffusion perturbed process have been used for the surplus of an insurance company which facing non-existence of reinsurance were Cramer-Lundberg model (classical risk process):

$$U_t = u + ct + \sigma W_t - \sum_{i=1}^{N_t} X_i \, , t \geq 0, \qquad \textit{Eq. 4}$$

where $u = U_0 \geq 0$ is the initial assets of the company and premium ratio is $c = (1+\theta)\lambda\mu > 0$. In this case, θ is the safety load of the insurer, $\{N_t\}$ is a homogeneous Poisson process with $\lambda > 0$ of intensity, and $\{X_i\}$ is an independent and identical distribution sequence of rigidly positive random variable with distribution function F. Then, for claim arrival process $\{N_t\}$ and $\{X_i\}$, they were assumed as independent. $\{W_t\}$ here is a standard one-dimensional Brownian motion independent of the compound Poisson process of S_t . Assume that $\mathbb{E}[X_i] = \mu < \infty$ and $F(0) = 0$. σW_t . The diffusion term is expressed as the change related to the insurer's profit at time t..

Reinsurance is consistently referred to risk management mechanism in the actuarial literature. At this time, reinsurance was considered cheap and the reinsurer and the insurer would pay the same premium. The quota share retention level be $k \in [0,1]$. Then the total claims of the insurer minus the reinsurance share of the quota is kX. If an insurer decides to purchase excess loss reinsurance at a retention rate $a \in [0, \infty)$ when the insurer's total claims are unprofitable, then $kX^\wedge a$ is given, minus quota and excess loss reinsurance. In this case, $\bar{R}$ is a reinsurance strategy that combines quota reinsurance and excess-of-loss reinsurance. The insurer's controlled surplus process then becomes

$$U_t^{\bar{R}} = u + c^{\bar{R}} t + \sigma W_t - \sum_{i=1}^{N_t} kX_i \wedge a, \qquad \textit{Eq. 5}$$

where the premium of the insurance is $c^{\bar{R}} = c - (1+\theta)\lambda\mathbb{E}[(kX_i - a)^+]$. And it has dynamics

$$dU_t^{\bar{R}} = c^{\bar{R}} dt + \sigma dW_t - d(\sum_{i=1}^{N_t} k\, X_i \wedge a), \qquad \textit{Eq. 6}$$

Time of ruin is then defined as $\tau^{\bar{R}} = \inf\{t \geq 0 | U_t^{\bar{R}} < 0\}$ and the ultimate ruin possibility is defined as $\psi^{\bar{R}} = \mathbb{P}(U_t^{\bar{R}} < 0 \text{ for some } t > 0)$. The reinsurance, $\bar{R}$ is called sustainable if $k \in [0,1]$ and $a \in [0, \infty)$. The purpose is to find the quota level k and the excess loss retention limit a to minimize the risk of the reinsurer or to maximize survival probability. At odds level $k = 1$, it is a pure excess of the Loss Reinsurance Treaty. If $c \geq (1+\theta)\lambda\mathbb{E}[(kX_i - a)^+]$, the insurance premium income is not negative. Therefore, is the XL storage level for which the equation $c = (1+\theta)\lambda\mathbb{E}[(kX_i - \underline{a})^+]$ holds.

The value function then defined as

$$\psi^{\bar{R}}(u) = \mathbb{P}\left(U_t \leq 0 \text{ for some } t \geq 0 \middle| U_0^{\bar{R}} = u\right) = \mathbb{P}(\tau^{\bar{R}} < \infty | U_0^{\bar{R}} = u) \quad \textit{Eq. 7}$$

where $\psi^{\bar{R}}(u)$ is the probability of final ruin of the portfolio policy $\bar{R}$ when the capital surplus is u. Now, minimize the probability of ruin is possible as follows:

$$\psi(u) = \min_{(k,a)\in\mathfrak{R}} \psi^{\bar{R}}(u) \quad \textit{Eq. 8}$$

where $\mathfrak{R}$ is sets of policies of the reinsurance.

Further, values of k^* and a^* were able to be obtained where could maximize the probability of ultimate survival $\phi(u) = 1 - \psi(u)$, hence we have optimal value function as follows:

$$\phi(u) = \max_{(k,a)\in\mathfrak{R}} \phi^{\bar{R}}(u).$$

2.3 Exponential distribution

For all notations and assumptions listed below were come from Jasiulewicz [2].

1. The total loss for a unit period $(n-1, n]$ were expressed as Z_n. Calculated at the end of each period. $\{Z_n, n = 1, 2, ...\}$ is an independent sequence assumed. Use the common distribution function $W(z)$ to trade random variables with the same distribution.
2. Premiums are calculated according to the principle of expected value with load factor $\theta > 0$. At the end of each unit period $(n-1, n]$ constant premium $c = (1+\theta)EZ_n$ is then paid.
3. U_n indicates the insurer's surplus at time n and it is calculated after payment. Next, at the beginning of the period $(n-1, n]$ was invested at random rate I_n.
4. $\{I_n, n = 0, 1, ...\}$ that also known as interest rate were assumed follow a time-homogeneous Markov chain. And further assume that for all $n = 0, 1, ...$ the rate I_n takes possible values $i_1, i_2, \dots, i_l$. The transition probability was denoted by
$$\Pr(I_{n+1} = i_t | I_n = i_s) = P_{st} \geq 0$$
And the initial distribution is denoted by
$$\Pr(I_0 = i_s) = \pi_s$$

3. Results and Discussion

$\Psi^b(u, i_s)$ calculation was conducted for the values of $b = 0.2, 0.3, \dots, 1.0$, $u = 1, 2, 3, 4, 5$ and $n = 1, 2, \dots, 10$. In this case, we considered

1. $l = 1$ for $i_1 = 0$.
2. $l = 2$ for $i_1 = 0.3$, $i_2 = 0.5$ and transition matrix

$$P = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}$$

And the value $\eta = 0.25$ and $\theta = 0.2$. For $Eh(Z_n, b) = b$. From *Eq.8*, the formula below were obtained

$$c(b) = (1+\eta)b - (\eta - \theta)$$

$$= 1.25b - 0.05$$

Then after condition were fulfilled, $b > 1 - \frac{\theta}{\eta} = 0.2$.

3.1 Exponential distribution

For the first step, Z_n were assumed to be an exponential distribution with mean equal to 1. Then the distribution function of $Z_n^{ce} = bZ_n$ is

$$V(x) = 1 - e^{-\frac{x}{b}} \qquad \textit{Eq. 9}$$

for $x \geq 0$ and $EZ_n^{ce} = b$. And $Var\ Z_n^{ce} = b^2$.

The ultimate ruin probability for finite time, $\Psi_n^b(u, i_s)$ were calculated by Markov Chain by fulfilling the form of

$$\Psi_n^b(u, i_s) \leq \left(1 - bR(b)\right) \sum_{t=1}^{l} P_{st} e^{-R(b)u(1+i_t)}, \quad n = 1, 2, \ldots$$

By substituting all of the value, Table 1 can assist on conclusion.

Table 3.1.1: The ruin probability for exponential distribution.

n	i_s	u	b = 0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5	3%	1	0.0087	0.0385	0.0776	0.1164	0.1512	0.1814	0.2073	0.2299	0.2494
		2	0.0001	0.0029	0.0119	0.0271	0.0460	0.0665	0.0871	0.1074	0.1265
		3	0.0000	0.0002	0.0017	0.0060	0.0134	0.0236	0.0357	0.0491	0.0630
		4	0.0000	0.0000	0.0002	0.0013	0.0038	0.0081	0.0143	0.0220	0.0308
		5	0.0000	0.0000	0.0000	0.0003	0.0010	0.0027	0.0056	0.0096	0.0148
	5%	1	0.0046	0.0256	0.0580	0.0934	0.1267	0.1568	0.1832	0.2067	0.2271
		2	0.0001	0.0015	0.0077	0.0196	0.0357	0.0542	0.0734	0.0927	0.1113
		3	0.0000	0.0001	0.0010	0.0040	0.0098	0.0183	0.0288	0.0409	0.0539
		4	0.0000	0.0000	0.0001	0.0008	0.0026	0.0061	0.0111	0.0178	0.0257
		5	0.0000	0.0000	0.0000	0.0002	0.0007	0.0020	0.0042	0.0076	0.0121
10	3%	1	0.0112	0.0493	0.0978	0.1448	0.1856	0.2203	0.2494	0.2749	0.2964
		2	0.0003	0.0049	0.0190	0.0411	0.0669	0.0936	0.1193	0.1442	0.1669
		3	0.0000	0.0005	0.0035	0.0113	0.0236	0.0391	0.0564	0.0748	0.0932
		4	0.0000	0.0000	0.0006	0.0030	0.0081	0.0160	0.0262	0.0383	0.0515
		5	0.0000	0.0000	0.0001	0.0008	0.0027	0.0064	0.0119	0.0193	0.0281
	5%	1	0.0049	0.0282	0.0654	0.1064	0.1452	0.1800	0.2103	0.2372	0.2605
		2	0.0001	0.0020	0.0101	0.0256	0.0462	0.0695	0.0932	0.1168	0.1392
		3	0.0000	0.0001	0.0015	0.0061	0.0146	0.0267	0.0411	0.0574	0.0742
		4	0.0000	0.0000	0.0002	0.0014	0.0046	0.0101	0.0180	0.0280	0.0393
		5	0.0000	0.0000	0.0000	0.0003	0.0014	0.0038	0.0078	0.0135	0.0206

The greater the u, the smaller the ruin probability.

At $n = 5$ and $n = 10$ time horizons, interest rate $I_0 = i_s = 0.03$ and 0.05 for every b and the ruin probability does not exceed 0.5 will produce the data that contain in Table 2.

Table 3.1.2: The ruin probability does not exceed 0.05 for exponential distribution and the maximal retention level *b*

Initial capital u		1	2	3	4	5
$n = 5$	$i_s = 3\%$	0.3289	0.6188	0.9062	1.0000	1.0000
	$i_s = 5\%$	0.3752	0.6775	0.9700	1.0000	1.0000
$n = 10$	$i_s = 3\%$	0.3005	0.5339	0.7626	0.9876	1.0000
	$i_s = 5\%$	0.3585	0.6160	0.8549	1.0000	1.0000

By referring to Table 3.1.2, the data could assist on concluding that company without reinsurance will exposed to below 5% level of bankruptcy.

3.2 Pareto distribution

Table 3.2.1: The ruin probability for Pareto distribution.

n	i_s	u	b = 0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5	3%	1	0.0471	0.0663	0.0818	0.0945	0.1050	0.1156	0.1214	0.1280	0.1337
		2	0.0255	0.0384	0.0499	0.0602	0.0693	0.0787	0.0846	0.0912	0.0972
		3	0.0169	0.0263	0.0352	0.0434	0.0510	0.0590	0.0644	0.0704	0.0759
		4	0.0124	0.0197	0.0267	0.0335	0.0399	0.0468	0.0515	0.0569	0.0618
		5	0.0097	0.0156	0.0214	0.0270	0.0325	0.0384	0.0427	0.0474	0.0519
	5%	1	0.0421	0.0603	0.0754	0.0880	0.0986	0.1092	0.1154	0.1222	0.1281
		2	0.0234	0.0356	0.0466	0.0566	0.0655	0.0748	0.0807	0.0873	0.0933
		3	0.0158	0.0247	0.0332	0.0411	0.0485	0.0563	0.0617	0.0676	0.0730
		4	0.0118	0.0187	0.0254	0.0320	0.0382	0.0448	0.0495	0.0548	0.0597
		5	0.0092	0.0148	0.0204	0.0259	0.0312	0.0370	0.0411	0.0458	0.0502
10	3%	1	0.0685	0.0947	0.1150	0.1312	0.1442	0.1599	0.1640	0.1718	0.1785
		2	0.0405	0.0599	0.0765	0.0907	0.1029	0.1175	0.1226	0.1309	0.1382
		3	0.0282	0.0432	0.0568	0.0690	0.0798	0.0929	0.0980	0.1060	0.1131
		4	0.0213	0.0334	0.0448	0.0552	0.0648	0.0765	0.0814	0.0888	0.0956
		5	0.0170	0.0270	0.0367	0.0458	0.0542	0.0647	0.0694	0.0762	0.0826
	5%	1	0.0582	0.0829	0.1028	0.1191	0.1325	0.1480	0.1532	0.1615	0.1687
		2	0.0354	0.0533	0.0691	0.0829	0.0949	0.1091	0.1148	0.1232	0.1307
		3	0.0252	0.0391	0.0519	0.0635	0.0740	0.0866	0.0921	0.1000	0.1072
		4	0.0194	0.0306	0.0413	0.0513	0.0605	0.0717	0.0768	0.0841	0.0908
		5	0.0156	0.0250	0.0341	0.0427	0.0509	0.0609	0.0656	0.0723	0.0786

Table 3.2.2: The ruin probability does not exceed 0.05 for Pareto distribution and the maximal retention level *b*

Initial capital *u*		1	2	3	4	5
$n = 5$	$i_s = 3\%$	0.2190	0.4052	0.5907	0.7696	0.9621
	$i_s = 5\%$	0.2468	0.4379	0.6209	0.8133	0.9996
$n = 10$	$i_s = 3\%$	lack	0.2567	0.3582	0.4588	0.5588
	$i_s = 5\%$	lack	0.2884	0.3933	0.4958	0.5974

By referring to the Table 3.2.2, the word 'lack' is representing the ruin probability exceeded 0.05 for ten-year-time horizon with initial capital u = 1.

4. Conclusion

For exponential distribution, if the initial capital grows, the part of the retained loss also grows with the constant level of risk of the company bankruptcy for any time horizon. The ideal degree of retention in a continuous risk process can be obtained by maximizing an adjustment coefficient relative to the level of retention. The aforementioned assertion is false in the discrete risk process.

The likelihood of ruin increases when the retention level b is increased for a fixed beginning capital u 1. As a result, if the retention level is low, the likelihood of disaster is low. It indicates that an insurer keeps only very little losses, resulting in extremely low revenue and a highly adverse situation for him. The correct strategy appears to be based on establishing an acceptable level of ruin probability and then determining the retention level in accordance with that likelihood.

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