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Analyzing the Efficiency of the Trapezoidal Rule and Simpson's Rule in Numerical Integration

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Abstract

This paper focuses on the numerical integration that aimed to compare the rate of performance or the rate of accuracy of the trapezoidal rule (TR) and Simpson's rule (SR). The objective of this project is estimate values of definite integral by using the TR and SR and to compare accuracy rate against these two methods. The three equations employed in this study will be used in Maple and Matlab software, and the results will be obtained once the equations have been numerically solved. A comparison between the numerical solutions of the TR and SR is also possible because of the numerical solution analysis results. In the event where the division condition is only even, SR provides a lower error number than other ways, while other approaches provide less precision. As a result, the Simpson technique is the most trustworthy approach for computing definite integrals and is more accurate than the TR. The error value of the data is based on the difference between exact and approximate values. The result of this paper is shown that SR is to contribute more precise correct value, rather than TR. However, Maple's software is the better instrument to consume for any mathematical problem solving that needs more than six decimal places, instead of Matlab's software to use in future work such as on the application of autopilot.

1. Introduction

This paper focuses on the numerical integration that aimed to compare the rate of performance or the rate of accuracy of the Trapezoidal rule (TR) and Simpson's rule (SR). This study will determine the different numerical methods of definite integrals which is the TR and SR. Other than that, this topic estimates which one value is aimed at comparing the rate of performance or the rate of accuracy of the TR and SR by using two different software which are Maple's software and Matlab software.

The total value or sum of $f(x)dx$ over the range from a to b is known as integration. Numerical integration is the process of estimating the value of a definite integral from the approximate numerical values of the integrand. Quadrature is the term for a function of a single variable that is used in numerical integration and that expresses the area under the curve $f(x)dx$. Numerical integration also includes a boarding family of algorithms for the purpose of counting the numerical values of a definite integral, and there are no singularities of the integrand in the domain under the assumption. These days, it is crucial because computers can integrate data in analytic ways as well, connecting analytical models to computer processors. A Course in Interpolation and Numeric Integration for the Mathematical Laboratory, both written by David Gibb in 1915, is where the phrase "numerical integration" was first used. Numerous fields, such as applied mathematics, statistics, economics, and engineering, are among the applications of numerical integration [2].

Numerical methods in integration constitute a broad family of algorithms for calculating the numerical value of a definite integral, and by extension, the term is also sometimes used to describe the numerical solution of differential equations like TR and SR. One common problem encountered with both the TR and SR is the issue of accurately approximating the integral when dealing with functions that have sharp spikes or discontinuities. Both methods may struggle to provide precise estimates in such cases, leading to potential inaccuracies in the result. Develop a computational study to compare the accuracy and efficiency of the TR and SR in approximating definite integrals. Consider functions with varying complexities and analyze the trade-offs between accuracy, computational effort, and convergence rates of the two numerical integration method.

This study will determine the different numerical methods of definite integrals which is the TR and SR. Other than that, this topic estimates which one value is aimed at comparing the rate of performance or the rate of accuracy of the TR and SR. The objective of this project is to estimate values of definite integral by using the TR and SR and to compare accuracy rate against these two methods. The scope of the study for this case study is analysis of trapezoidal using Maple and Matlab software to know the answer from the equation. Next, analysis of SR using Maple and Matlab to know the answer from the equation. Moreover, the determination of the most accurate rule to get the result from the answer that gets. Lastly, determination of the best-solving problem between two software to get the precise result.

2. Methodology

All the information required to produce the project's results is described in the technique part, which is often known as the materials and methods section. The results will be collected by using the software Maple and Matlab. The aims to gain the final answer, and the equation will be coded into both software. The Trapezoidal and SR procedures will apply in this software. Following the completion of all findings, it will be decided how to continue based on the accuracy and relative inaccuracy. There are formulas that is used in this research which are;

SR:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a)+f(b)+4\sum_{\substack{i=1\\ i\ odd}}^{n-1} f(a+ih)+2\sum_{\substack{i=1\\ i\ even}}^{n-1} f(a+ih)\right],$$

and TR:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a)+f(b)+2\sum_{i=1}^{n-1} f(a+ih)\right].$$

There are three equations will be used in this project as shown in Eq. (1), Eq. (2), and Eq. (3). The Maple software will be used to gather the results. The equation will be coded into Maple to get the definitive response. In this software, the Trapezoidal and SR procedures will be used. Depending on the accuracy and relative inaccuracy of all findings, a course of action will be chosen. The following three problematic equations will be applied in this research about the comparison of Trapezoidal and SR:

$$f(x) = \int_0^4 \frac{x^3}{\sqrt{x^2+9}}\,dx. \tag{1}$$

$$f(x) = \int_0^\pi \frac{dx}{(2+\cos x)}. \tag{2}$$

$$f(x) = \int_0^1 x^2 e^{x^3}\,dx. \tag{3}$$

2.1 Numerical Solutions

Eq. (1) until Eq. (3) were applied in both software, which are Maple and Matlab. Both software will be running to solve using TR and SR 1/3. The code that was learned by the book will apply [2][3][4]. Each TR and SR needs to use subinterval, *n*. In this problem, 10, 20, 30, 60, 100 and 200 will be used as subinterval. After all the coding has been written, it will give the value of approximate of all three equations. Other than that, it also will give the value of exact solution and error value of approximate.

All the final answer for the TR and SR 1/3 then will be transfer to the table. From the error result, the graph will plot to make a comparison for both rules to determine which numerical solution are more accurate compared to exact values.

3. Result and Discussion

The raw data for this study was fully collected and assembled into a table and graph. The results and discussion section presents data and analysis of the study. This section can be organized based on the stated objectives. Results can be presented in the form of tables of comparison between two rules, comparison between two software that was used, and table of differences of error.

Table 1: *Data of comparison between two rules*

	n	Exact value	TR	Error	SR	Error
First equation	10	14.66666667	14.76743474	0.10076807	14.66636247	0.0003042
	20	14.66666667	14.69184466	0.02517799	14.66664796	0.00001871
	30	14.66666667	14.67785573	0.01118906	14.66666298	$3.69*10^{-6}$
	60	14.66666667	14.66946376	0.00279709	14.66666643	$2.4*10^{-7}$
	100	14.66666667	14.66767361	0.00100694	14.66666664	$3.*10^{-8}$
	200	14.66666667	14.6669184	0.00025173	14.66666667	$1.*10^{-8}$
Second equation	10	1.813799365	1.813799364	$1.*10^{-9}$	1.813797058	$2.307*10^{-6}$
	20	1.813799365	1.813799365	0	1.813799365	0
	30	1.813799365	1.813799365	0	1.813799364	$1.*10^{-9}$
	60	1.813799365	1.813799364	$1.*10^{-9}$	1.813799365	0
	100	1.813799365	1.813799364	$1.*10^{-9}$	1.813799364	$1.*10^{-9}$
	200	1.813799365	1.813799365	0	1.813799365	0
Third equation	10	0.572760609	0.584014214	0.011253605	0.573035358	0.000274749
	20	0.572760609	0.575587569	0.002826959	0.572778687	$1.80773*10^{-5}$
	30	0.572760609	0.574018166	0.001257556	0.572764216	$3.6065*10^{-6}$
	60	0.572760609	0.573075169	0.00031456	0.572760836	$2.268*10^{-7}$
	100	0.572760609	0.572873864	0.000113255	0.572760639	$2.94*10^{-8}$
	200	0.572760609	0.572788925	$2.83153*10^{-5}$	0.572760611	$2.1*10^{-9}$

Table 1 shows the exact value and the estimated value with error obtained from TR and SR for three equations with different values of *n* by using Maple Software. From this table, all three equations give the value of SR less error compared with TR for each *n* subinterval and the estimated value becomes more accurate if the value of *n* is bigger.

Table 2: *Data of comparison of approximate value between two software*

	Rules	n	Exact	Maple	Matlab
First equation	TR	10	14.66666667	14.76743474	14.767435
		20	14.66666667	14.69184466	14.691845
		30	14.66666667	14.67785573	14.677856
		60	14.66666667	14.66946376	14.669464
		100	14.66666667	14.66767361	14.667674
		200	14.66666667	14.6669184	14.666918
	SR	10	14.66666667	14.66636247	14.666362
		20	14.66666667	14.66664796	14.666648
		30	14.66666667	14.66666298	14.666663
		60	14.66666667	14.66666643	14.666666
		100	14.66666667	14.66666664	14.666667
		200	14.66666667	14.66666667	14.666667

Second equation	TR	10	1.813799365	1.813799364	1.813799
		20	1.813799365	1.813799365	1.813799
		30	1.813799365	1.813799365	1.813799
		60	1.813799365	1.813799364	1.813799
		100	1.813799365	1.813799364	1.813799
		200	1.813799365	1.813799365	1.813799
	SR	10	1.813799365	1.813797058	2.58779
		20	1.813799365	1.813799365	2.587762
		30	1.813799365	1.813799364	2.587757
		60	1.813799365	1.813799365	2.587755
		100	1.813799365	1.813799364	2.587755
		200	1.813799365	1.813799365	2.587755
Third equation	TR	10	0.572760609	0.584014214	0.584014
		20	0.572760609	0.575587569	0.575588
		30	0.572760609	0.574018166	0.574018
		60	0.572760609	0.573075169	0.573075
		100	0.572760609	0.572873864	0.572874
		200	0.572760609	0.572788925	0.572789
	SR	10	0.572760609	0.573035358	0.392441
		20	0.572760609	0.572778687	0.462186
		30	0.572760609	0.572764216	0.493826
		60	0.572760609	0.572760836	0.530484
		100	0.572760609	0.572760639	0.546685
		200	0.572760609	0.572760611	0.559449

Table 2 shows the results for all three equation that has the same value for the exact and approximate value between two different software. Thus, there have a small difference at the approximate value based on decimal place. The Maple software indicates that the approximate value was given in nine decimal places. On the other hand, Matlab software is only given in six decimal places. For analysis based on the result from Maple and Matlab Software, Matlab are given the oblation value while Maple is not.

Table 3: *Data of difference error*

	n	Exact	Maple	Matlab
TR	10	0.572760609	0.011253605	0.011254
	20	0.572760609	0.002826959	0.002827
	30	0.572760609	0.001257556	0.001258
	60	0.572760609	0.00031456	0.000315
	100	0.572760609	0.000113255	0.000113
	200	0.572760609	$2.83153*10^{-5}$	0.000028
SR	10	0.572760609	0.000274749	0.180319
	20	0.572760609	$1.80773*10^{-5}$	0.110574
	30	0.572760609	$3.6065*10^{-6}$	0.078935
	60	0.572760609	$2.268*10^{-7}$	0.042276
	100	0.572760609	$2.94*10^{-8}$	0.026076
	200	0.572760609	$2.1*10^{-9}$	0.013312

To achieve the second objective, it can be shown in Table 3. The error is obtained from the difference between the estimated and exact value. Regarding the error that was obtained from the third equation, there are two conditions to determine the difference of the error value which are based on the difference in software and the differences rules. The difference error that indicates by Maple is more precise compared to Matlab due to the existence of nine decimal places. For the best method between TR and SR, the SR provided a lower error number. This error also can be present in the form of graph as shown in Fig. 1.

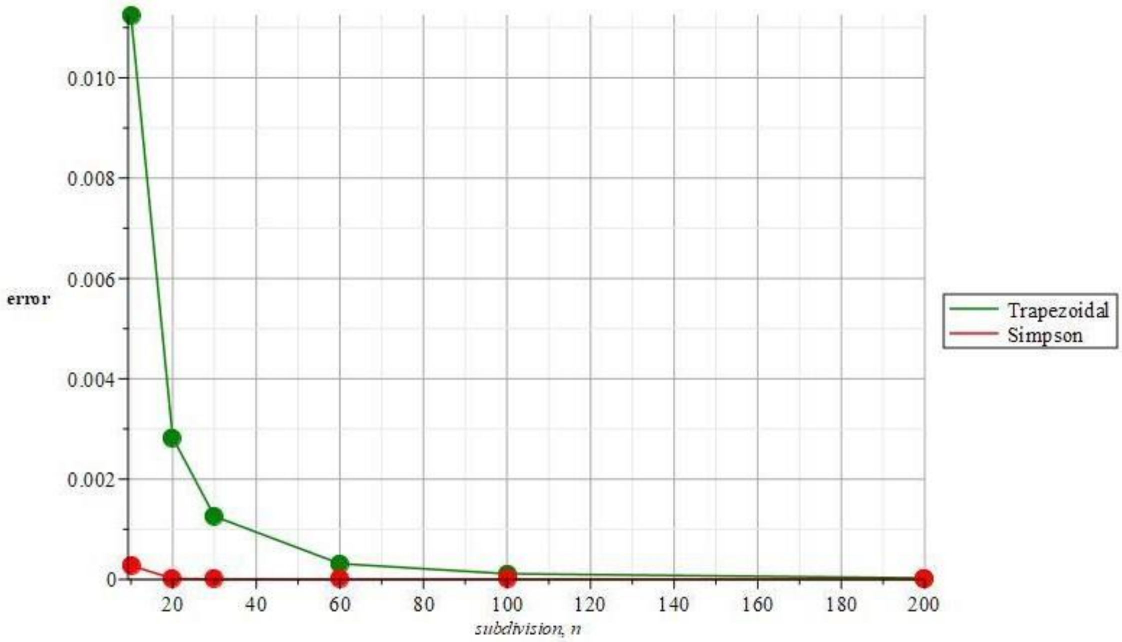

Fig. 1 *Error between TR and SR*

4. Conclusions

In conclusion, the numerical method, is a method for solving mathematical models with solving techniques that are mathematically formulated with basic arithmetic operations and are repeated manually or with the aid of computers, including the TR method. The results obtained from this method take the form of approximations that lead to errors. If the integral cannot be solved analytically, one may use this numerical integral method [6].

However, A large family of algorithms for determining the numerical value of a definite integral makes up numerical integration. Numerical integration is the most common method of finding the solution because some integration problems cannot be solved analytically. To address numerical integration for uneven data space, numerous techniques are applied and used. Numerical integration issues are frequently resolved using the TR and SR [7].

SR provides more accurate approximations for integral calculations compared to the Trapezoidal Rule. This is because Simpson's Rule utilizes quadratic approximations, which can better capture the curvature of the function being integrated, resulting in more precise estimates. However, SR may require more function evaluations than the TR, making it computationally more expensive in some cases. Ultimately, the choice between the two methods depends on the specific problem at hand and the desired trade-off between accuracy and computational efficiency.

To sum up, this research had been succeeded of studies between TR and SR. The result shows that SR is the most accurate rather than TR. Furthermore, these results are in line with the results expected at the beginning of this research. The Maple's software is recommended for the solving-problem works rather than Matlab's software.

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Conflict of Interest

Authors declare that there is no conflict of interests regarding the publication of the paper.

Author Contribution

The authors confirm contribution to the paper as follows: ***study conception and design:*** *Nur Aainaa Athirah Ali, Muhammad Alif Abdul Talib, Zul Afiq Sazeli;* ***data collection:*** *Nur Aainaa Athirah Ali, Muhammad Alif Abdul Talib, Zul Afiq Sazeli;* ***analysis and interpretation of results:*** *Nur Aainaa Athirah Ali, Muhammad Alif Abdul Talib, Zul Afiq Sazeli;* ***draft manuscript preparation:*** *Nur Aainaa Athirah Ali, Muhammad Alif Abdul Talib, Zul Afiq Sazeli. All authors reviewed the results and approved the final version of the manuscript.*

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